

# ECE 312

# Electronic Circuits (A)

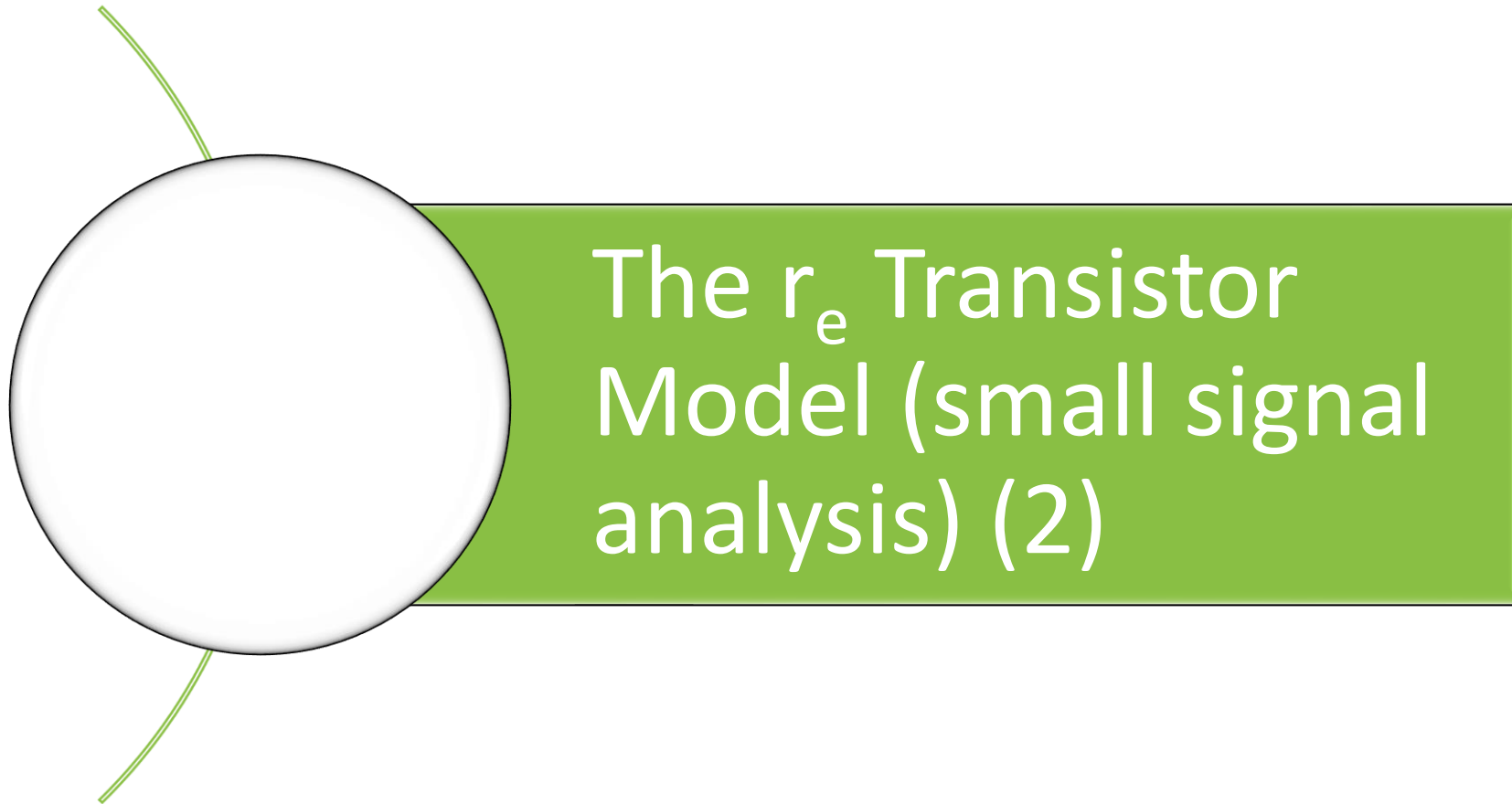
Lec. 7: BJT Modeling and re Transistor Model (small signal analysis) (2)

Instructor

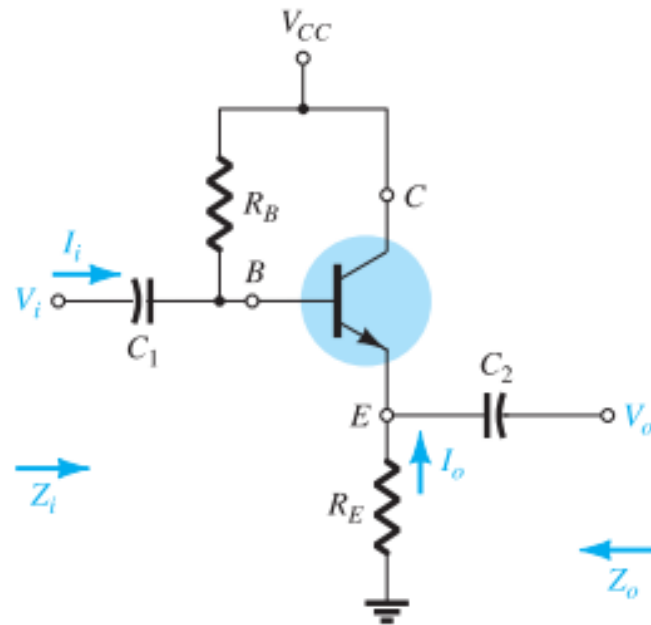
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# Agenda

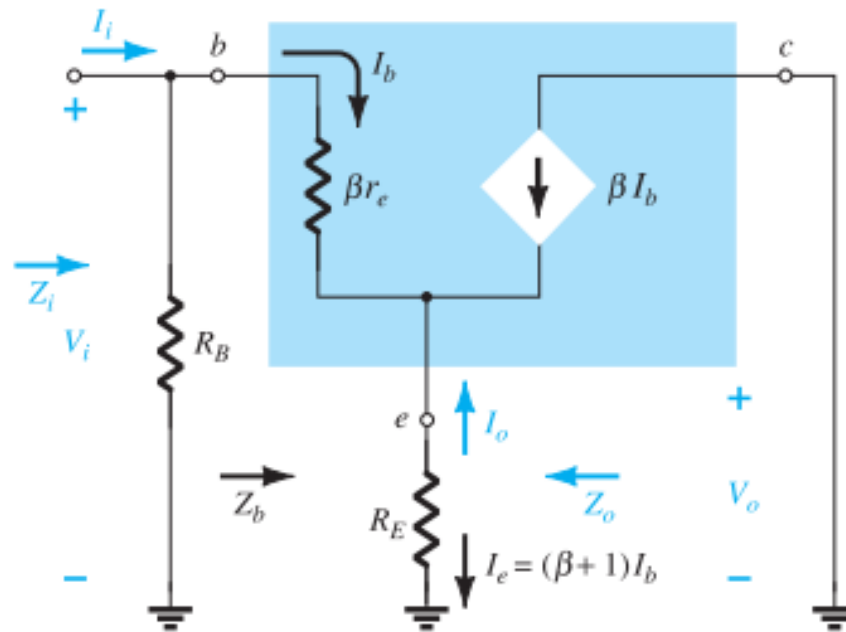


# Emitter Follower (Common Collector) Configuration



**FIG. 5.36**

*Emitter-follower configuration.*



**FIG. 5.37**

*Substituting the  $r_e$  equivalent circuit into the ac equivalent network of Fig. 5.36.*

$$Z_i = R_B \parallel Z_b$$

$$Z_b = \beta r_e + (\beta + 1)R_E$$

$$Z_b \cong \beta(r_e + R_E)$$

$$Z_b \cong \beta R_E \quad R_E \gg r_e$$

# Emitter Follower Configuration (o/p impedance and Gain)

$$I_e = (\beta + 1)I_b = (\beta + 1)\frac{V_i}{Z_b}$$

$$I_e = \frac{(\beta + 1)V_i}{\beta r_e + (\beta + 1)R_E}$$

$$I_e = \frac{V_i}{[\beta r_e / (\beta + 1)] + R_E}$$

$$(\beta + 1) \cong \beta$$

$$\frac{\beta r_e}{\beta + 1} \cong \frac{\beta r_e}{\beta} = r_e$$

$$I_e \cong \frac{V_i}{r_e + R_E}$$

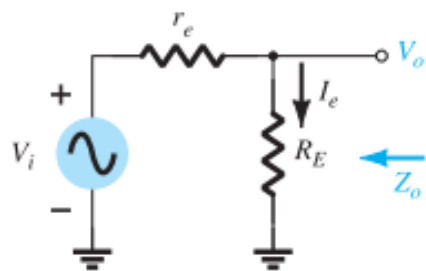


FIG. 5.38

Defining the output impedance for the emitter-follower configuration.

$$Z_o = R_E \parallel r_e$$

$$Z_o \cong r_e$$

$$V_o = \frac{R_E V_i}{R_E + r_e}$$

$$A_v = \frac{V_o}{V_i} = \frac{R_E}{R_E + r_e}$$

Because  $R_E$  is usually much greater than  $r_e$ ,

$$R_E + r_e \cong R_E;$$

$$A_v = \frac{V_o}{V_i} \cong 1$$

in phase

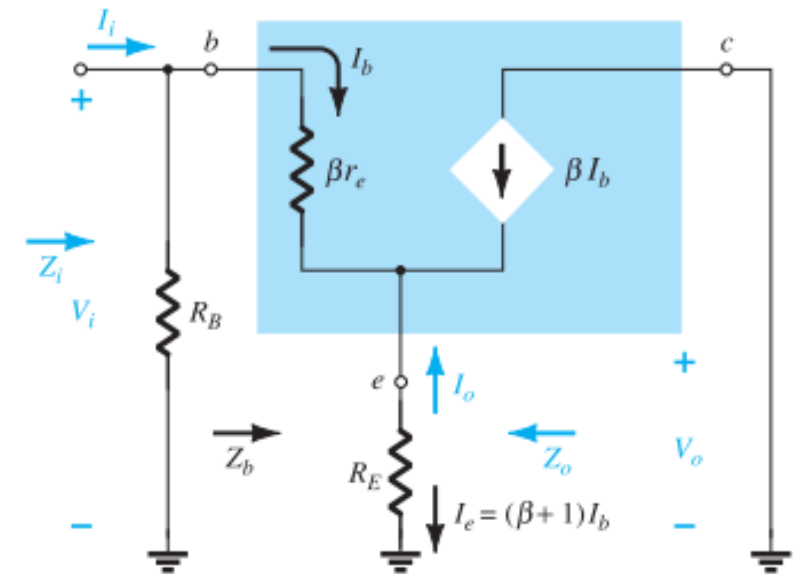


FIG. 5.37

Substituting the  $r_e$  equivalent circuit into the ac equivalent network of Fig. 5.36.

# Emitter Follower Configuration..

## Effect of $r_o$

$$Z_b = \beta r_e + \frac{(\beta + 1)R_E}{1 + \frac{R_E}{r_o}}$$

$$Z_o = r_o \parallel R_E \parallel \frac{\beta r_e}{(\beta + 1)}$$

$$A_v = \frac{(\beta + 1)R_E/Z_b}{1 + \frac{R_E}{r_o}}$$

$$r_o \geq 10R_E$$

$$Z_b = \beta r_e + (\beta + 1)R_E$$

$$Z_o = r_o \parallel R_E \parallel r_e$$

$$A_v \cong \frac{\beta R_E}{Z_b}$$

$$Z_b \cong \beta(r_e + R_E)$$

$r_o \geq 10R_E$

$$Z_o \cong R_E \parallel r_e$$

Any  $r_o$

$$Z_b \cong \beta(r_e + R_E)$$

$$A_v \cong \frac{\beta R_E}{\beta(r_e + R_E)}$$

$$Z_o \cong r_e$$

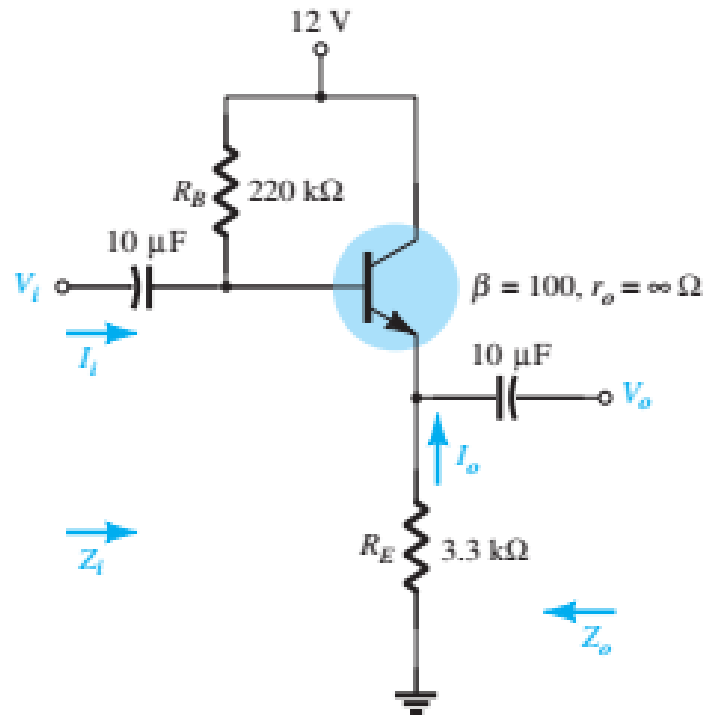
$$A_v \cong \frac{R_E}{r_e + R_E}$$

$r_o \geq 10R_E$

# Emitter Follower Configuration (Example (wo/ro))

**EXAMPLE 5.7** For the emitter-follower network of Fig. 5.39, determine:

- $r_e$ .
- $Z_i$ .
- $Z_o$ .
- $A_v$ .
- Repeat parts (b) through (d) with  $r_o = 25 \text{ k}\Omega$  and compare results.



**FIG. 5.39**

**Solution:**

$$\begin{aligned} \text{a. } I_B &= \frac{V_{CC} - V_{BE}}{R_B + (\beta + 1)R_E} \\ &= \frac{12 \text{ V} - 0.7 \text{ V}}{220 \text{ k}\Omega + (101)3.3 \text{ k}\Omega} = 20.42 \mu\text{A} \end{aligned}$$

$$\begin{aligned} I_E &= (\beta + 1)I_B \\ &= (101)(20.42 \mu\text{A}) = 2.062 \text{ mA} \end{aligned}$$

$$r_e = \frac{26 \text{ mV}}{I_E} = \frac{26 \text{ mV}}{2.062 \text{ mA}} = 12.61 \Omega$$

$$\begin{aligned} \text{b. } Z_b &= \beta r_e + (\beta + 1)R_E \\ &= (100)(12.61 \Omega) + (101)(3.3 \text{ k}\Omega) \\ &= 1.261 \text{ k}\Omega + 333.3 \text{ k}\Omega \\ &= 334.56 \text{ k}\Omega \cong \beta R_E \end{aligned}$$

$$\begin{aligned} Z_i &= R_B \parallel Z_b = 220 \text{ k}\Omega \parallel 334.56 \text{ k}\Omega \\ &= 132.72 \text{ k}\Omega \end{aligned}$$

$$\begin{aligned} \text{c. } Z_o &= R_E \parallel r_e = 3.3 \text{ k}\Omega \parallel 12.61 \Omega \\ &= 12.56 \Omega \cong r_e \end{aligned}$$

$$\begin{aligned} \text{d. } A_v &= \frac{V_o}{V_i} = \frac{R_E}{R_E + r_e} = \frac{3.3 \text{ k}\Omega}{3.3 \text{ k}\Omega + 12.61 \Omega} \\ &= 0.996 \cong 1 \end{aligned}$$

# Emitter Follower Configuration (Example w/ro)

e. Checking the condition  $r_o \geq 10R_E$ , we have

$$25 \text{ k}\Omega \geq 10(3.3 \text{ k}\Omega) = 33 \text{ k}\Omega$$

which is *not* satisfied. Therefore,

$$\begin{aligned} Z_b &= \beta r_e + \frac{(\beta + 1)R_E}{1 + \frac{R_E}{r_o}} = (100)(12.61 \text{ }\Omega) + \frac{(100 + 1)3.3 \text{ k}\Omega}{1 + \frac{3.3 \text{ k}\Omega}{25 \text{ k}\Omega}} \\ &= 1.261 \text{ k}\Omega + 294.43 \text{ k}\Omega \\ &= 295.7 \text{ k}\Omega \end{aligned}$$

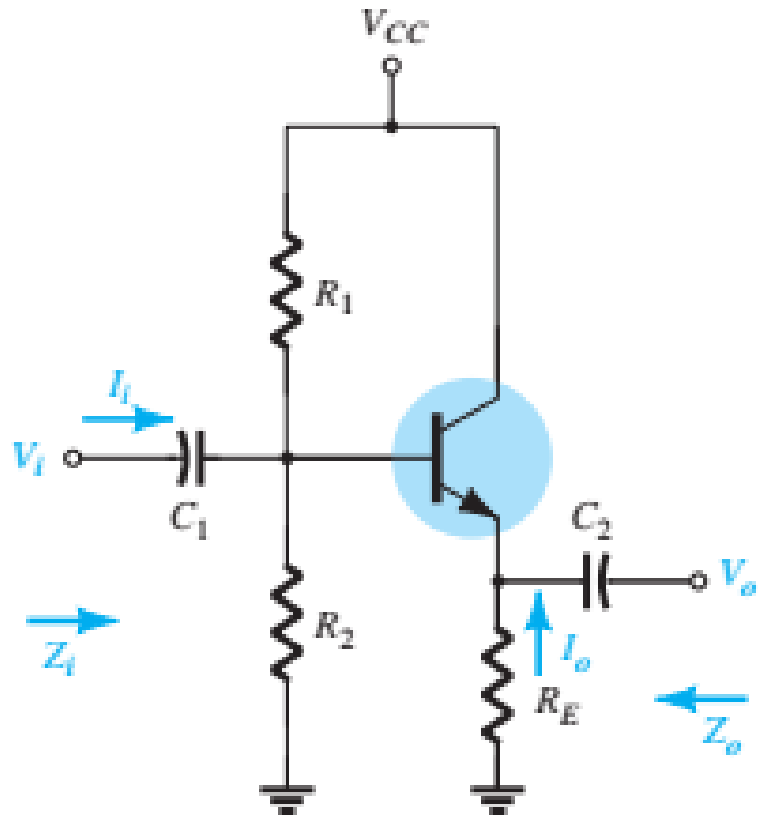
with  $Z_i = R_B \parallel Z_b = 220 \text{ k}\Omega \parallel 295.7 \text{ k}\Omega$   
 $= \mathbf{126.15 \text{ k}\Omega}$  vs.  $132.72 \text{ k}\Omega$  obtained earlier

$$Z_o = R_E \parallel r_e = \mathbf{12.56 \text{ }\Omega}$$
 as obtained earlier

$$\begin{aligned} A_v &= \frac{(\beta + 1)R_E/Z_b}{\left[1 + \frac{R_E}{r_o}\right]} = \frac{(100 + 1)(3.3 \text{ k}\Omega)/295.7 \text{ k}\Omega}{\left[1 + \frac{3.3 \text{ k}\Omega}{25 \text{ k}\Omega}\right]} \\ &= \mathbf{0.996 \cong 1} \end{aligned}$$

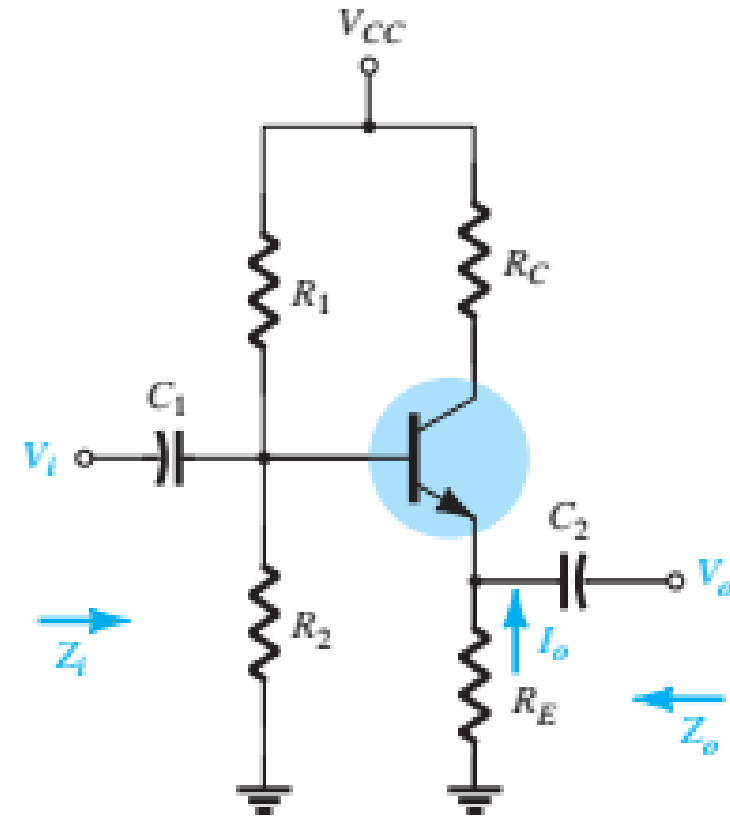
matching the earlier result.

# Emitter Follower Configuration (Diff. Biasing)



**FIG. 5.40**

*Emitter-follower configuration with a voltage-divider biasing arrangement.*

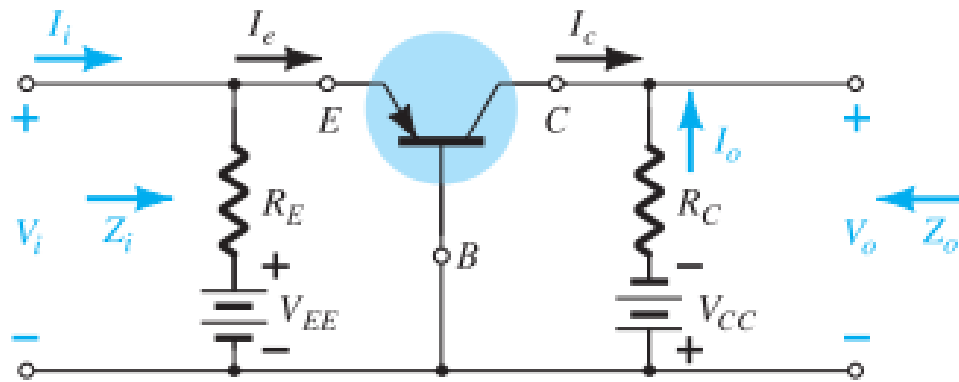


**FIG. 5.41**

*Emitter-follower configuration with a collector resistor  $R_C$ .*

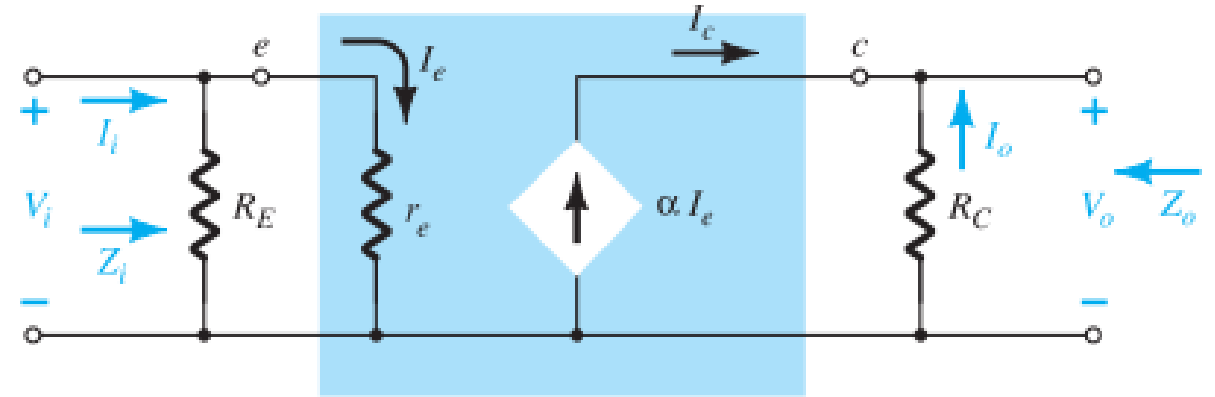


# Common-Base Configuration



**FIG. 5.42**

*Common-base configuration.*



**FIG. 5.43**

*Substituting the  $r_e$  equivalent circuit into the ac equivalent network of Fig. 5.44.*

$$Z_i = R_E \parallel r_e$$

$$I_e = I_i$$

$$I_o = -\alpha I_e = -\alpha I_i$$

$$Z_o = R_C$$

$$A_i = \frac{I_o}{I_i} = -\alpha \cong -1$$

$$V_o = -I_o R_C = -(-I_c) R_C = \alpha I_e R_C$$

$$I_e = \frac{V_i}{r_e}$$

$$V_o = \alpha \left( \frac{V_i}{r_e} \right) R_C$$

$$A_v = \frac{V_o}{V_i} = \frac{\alpha R_C}{r_e} \cong \frac{R_C}{r_e}$$

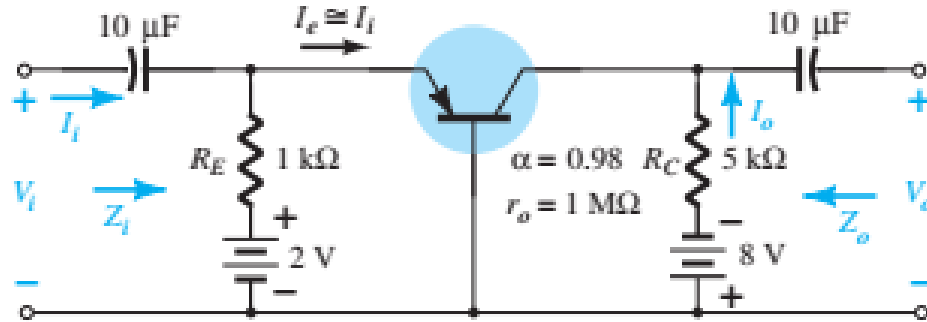
**Phase Relationship** The fact that  $A_v$  is a positive number shows that  $V_o$  and  $V_i$  are in phase for the common-base configuration.

**Effect of  $r_o$**  For the common-base configuration,  $r_o = 1/h_{ob}$  is typically in the megohm range and sufficiently larger than the parallel resistance  $R_C$  to permit the approximation  $r_o \parallel R_C \cong R_C$ .

# Common-Base Configuration (Example)

**EXAMPLE 5.8** For the network of Fig. 5.44, determine:

- $r_e$ .
- $Z_i$ .
- $Z_o$ .
- $A_v$ .
- $A_i$ .



**FIG. 5.** *Solution:*

$$\text{a. } I_E = \frac{V_{EE} - V_{BE}}{R_E} = \frac{2 \text{ V} - 0.7 \text{ V}}{1 \text{ k}\Omega} = \frac{1.3 \text{ V}}{1 \text{ k}\Omega} = 1.3 \text{ mA}$$

$$r_e = \frac{26 \text{ mV}}{I_E} = \frac{26 \text{ mV}}{1.3 \text{ mA}} = \mathbf{20 \Omega}$$

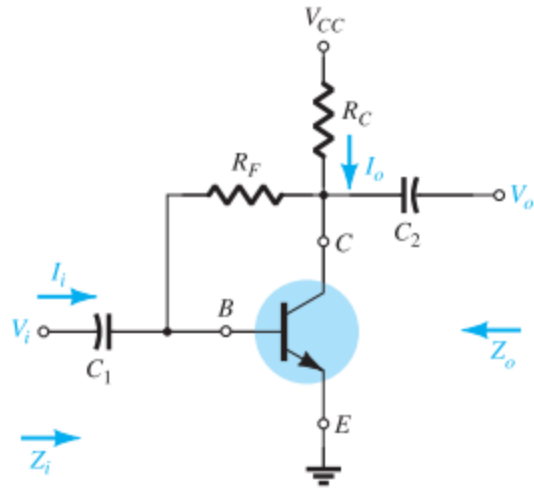
$$\begin{aligned} \text{b. } Z_i &= R_E \parallel r_e = 1 \text{ k}\Omega \parallel 20 \Omega \\ &= \mathbf{19.61 \Omega} \cong r_e \end{aligned}$$

$$\text{c. } Z_o = R_C = \mathbf{5 \text{ k}\Omega}$$

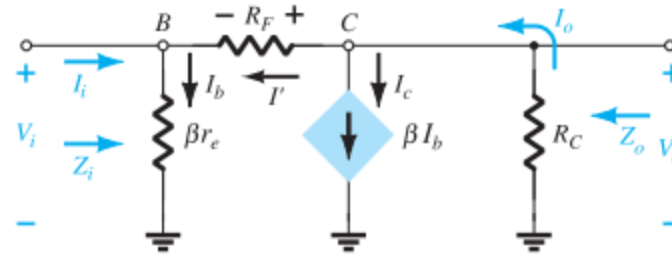
$$\text{d. } A_v \cong \frac{R_C}{r_e} = \frac{5 \text{ k}\Omega}{20 \Omega} = \mathbf{250}$$

$$\text{e. } A_i = \mathbf{-0.98} \cong -1$$

# Collector-Feedback Configuration



**FIG. 5.45**  
Collector feedback configuration.



**FIG. 5.46**  
Substituting the  $r_e$  equivalent circuit into the ac equivalent network of F

$$I_o = I' + \beta I_b$$

$$I' = \frac{V_o - V_i}{R_F}$$

$$V_o = -I_o R_C = -(I' + \beta I_b) R_C$$

$$V_i = I_b \beta r_e$$

$$I' = -\frac{(I' + \beta I_b) R_C - I_b \beta r_e}{R_F} = -\frac{I' R_C}{R_F} - \frac{\beta I_b R_C}{R_F} - \frac{I_b \beta r_e}{R_F}$$

$$I' \left( 1 + \frac{R_C}{R_F} \right) = -\beta I_b \frac{(R_C + r_e)}{R_F}$$

$$I' = -\beta I_b \frac{(R_C + r_e)}{R_C + R_F}$$

$$I_i = I_b - I' = I_b + \beta I_b \frac{(R_C + r_e)}{R_C + R_F}$$

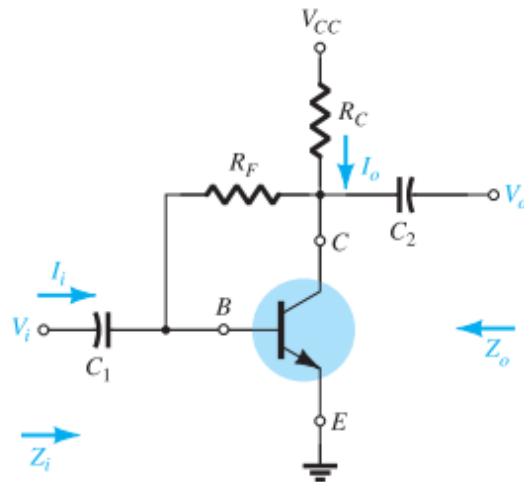
$$I_i = I_b \left( 1 + \beta \frac{(R_C + r_e)}{R_C + R_F} \right)$$

$$Z_i = \frac{V_i}{I_i} = \frac{I_b \beta r_e}{I_b \left( 1 + \beta \frac{(R_C + r_e)}{R_C + R_F} \right)} = \frac{\beta r_e}{1 + \beta \frac{(R_C + r_e)}{R_C + R_F}}$$

$$R_C \gg r_e \quad Z_i = \frac{\beta r_e}{1 + \frac{\beta R_C}{R_C + R_F}}$$

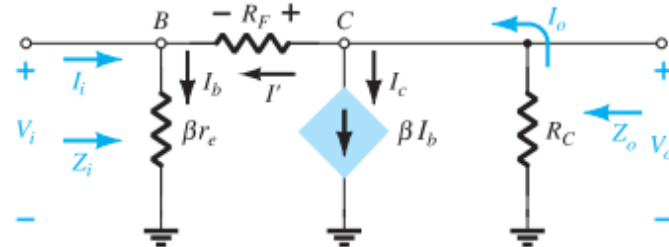
$$Z_i = \frac{r_e}{\frac{1}{\beta} + \frac{R_C}{R_C + R_F}}$$

# Collector-Feedback Configuration



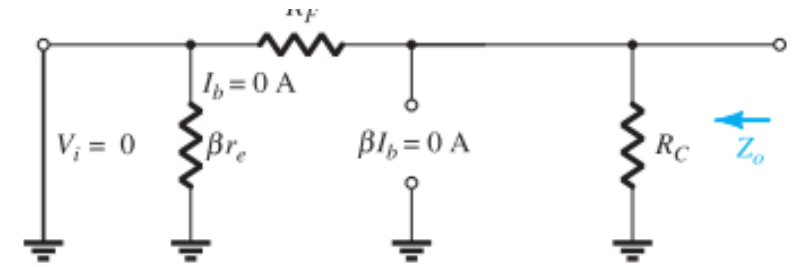
**FIG. 5.45**

Collector feedback configuration.



**FIG. 5.46**

Substituting the  $r_e$  equivalent circuit into the ac equivalent network of Fig. 5.45.



**FIG. 5.47**

Defining  $Z_o$  for the collector feedback configuration.

$$Z_o \cong R_C \parallel R_F$$

$$A_v = -\frac{(R_C + R_F - R_C)R_C}{R_C + R_F r_e}$$

$$A_v = -\left(\frac{R_F}{R_C + R_F}\right)\frac{R_C}{r_e}$$

$$A_v \cong -\frac{R_C}{r_e}$$

180° phase shift

$$\begin{aligned} V_o &= -I_o R_C = -(I' + \beta I_b) R_C \\ &= -\left(-\beta I_b \frac{(R_C + r_e)}{R_C + R_F} + \beta I_b\right) R_C \end{aligned}$$

$$V_o = -\beta I_b \left(1 - \frac{(R_C + r_e)}{R_C + R_F}\right) R_C$$

$$A_v = \frac{V_o}{V_i} = \frac{-\beta I_b \left(1 - \frac{(R_C + r_e)}{R_C + R_F}\right) R_C}{\beta r_e I_b}$$

$$= -\left(1 - \frac{(R_C + r_e)}{R_C + R_F}\right) \frac{R_C}{r_e}$$

$$A_v = -\left(1 - \frac{R_C}{R_C + R_F}\right) \frac{R_C}{r_e}$$

# Collector-Feedback Configuration..

## Effect of $r_o$

$$Z_i = \frac{1 + \frac{R_C \parallel r_o}{R_F}}{\frac{1}{\beta r_e} + \frac{1}{R_F} + \frac{R_C \parallel r_o}{\beta r_e R_F} + \frac{R_C \parallel r_o}{R_F r_e}}$$

$$r_o \geq 10R_C$$

$$Z_i = \frac{1 + \frac{R_C}{R_F}}{\frac{1}{\beta r_e} + \frac{1}{R_F} + \frac{R_C}{\beta r_e R_F} + \frac{R_C}{R_F r_e}} = \frac{r_e \left[ 1 + \frac{R_C}{R_F} \right]}{\frac{1}{\beta} + \frac{1}{R_F} \left[ r_e + \frac{R_C}{\beta} + R_C \right]}$$

Applying  $R_C \gg r_e$  and  $\frac{R_C}{\beta}$ ,

$$Z_i \cong \frac{r_e \left[ 1 + \frac{R_C}{R_F} \right]}{\frac{1}{\beta} + \frac{R_C}{R_F}} = \frac{r_e \left[ \frac{R_F + R_C}{R_F} \right]}{\frac{R_F + \beta R_C}{\beta R_F}} = \frac{r_e}{\frac{1}{\beta} \left( \frac{R_F}{R_F + R_C} \right) + \frac{R_C}{R_C + R_F}}$$

but, since  $R_F$  typically  $\gg R_C$ ,  $R_F + R_C \cong R_F$  and  $\frac{R_F}{R_F + R_C} = 1$

$$Z_i \cong \frac{r_e}{\frac{1}{\beta} + \frac{R_C}{R_C + R_F}} \quad r_o \gg R_C, R_F > R_C$$

$$Z_o = r_o \parallel R_C \parallel R_F$$

For  $r_o \geq 10R_C$ ,

$$Z_o \cong R_C \parallel R_F \quad r_o \geq 10R_C$$

$$Z_o \cong R_C \quad r_o \geq 10R_C, R_F \gg R_C$$

$$A_v = - \left( \frac{R_F}{R_C \parallel r_o + R_F} \right) \frac{R_C \parallel r_o}{r_e}$$

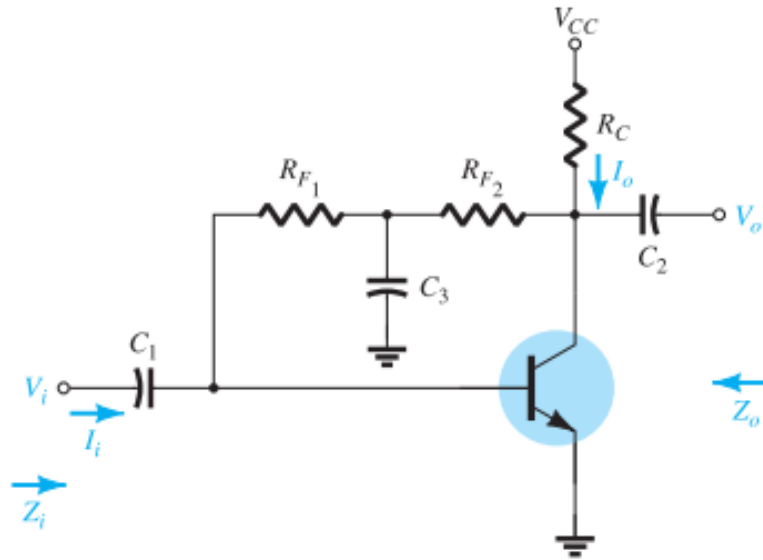
For  $r_o \geq 10R_C$ ,

$$A_v \cong - \left( \frac{R_F}{R_C + R_F} \right) \frac{R_C}{r_e} \quad r_o \geq 10R_C$$

and for  $R_F \gg R_C$

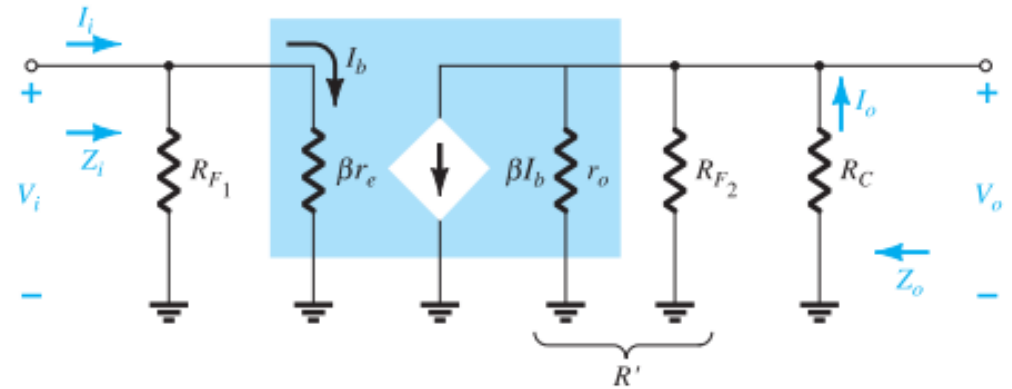
$$A_v \cong - \frac{R_C}{r_e} \quad r_o \geq 10R_C, R_F \geq R_C$$

# Collector DC Feedback Configuration



**FIG. 5.50**

Collector dc feedback configuration.



**FIG. 5.51**

Substituting the  $r_e$  equivalent circuit into the ac equivalent network of Fig. 5.50.

$$Z_i = R_{F1} \parallel \beta r_e$$

$$Z_o = R_C \parallel R_{F2} \parallel r_o$$

$$Z_o \cong R_C \parallel R_{F2} \quad r_o \geq 10R_C$$

$$R' = r_o \parallel R_{F2} \parallel R_C$$

$$V_o = -\beta I_b R'$$

$$I_b = \frac{V_i}{\beta r_e}$$

$$V_o = -\beta \frac{V_i}{\beta r_e} R'$$

$$A_v = \frac{V_o}{V_i} = -\frac{r_o \parallel R_{F2} \parallel R_C}{r_e}$$

$$A_v = \frac{V_o}{V_i} \cong -\frac{R_{F2} \parallel R_C}{r_e}$$

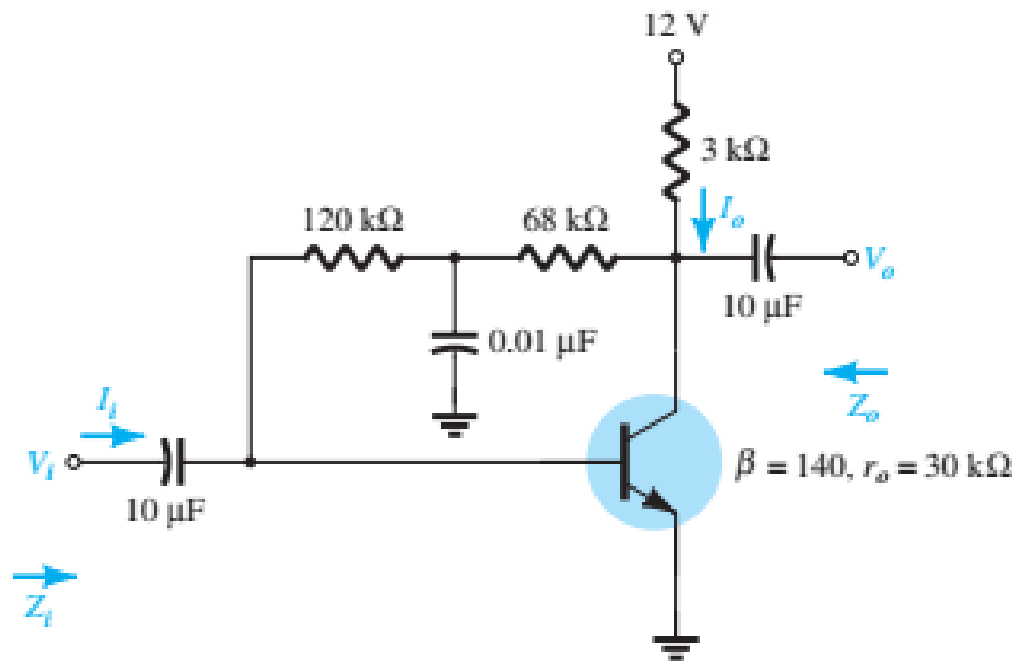
$r_o \geq 10R_C$

180° phase shift

# Collector DC Feedback Configuration (Example)

**EXAMPLE 5.10** For the network of Fig. 5.52, determine:

- $r_e$
- $Z_i$
- $Z_o$
- $A_v$
- $V_o$  if  $V_i = 2 \text{ mV}$



**FIG. 5.52**

**Solution:**

$$\begin{aligned} \text{a. DC: } I_B &= \frac{V_{CC} - V_{BE}}{R_F + \beta R_C} \\ &= \frac{12 \text{ V} - 0.7 \text{ V}}{(120 \text{ k}\Omega + 68 \text{ k}\Omega) + (140)3 \text{ k}\Omega} \\ &= \frac{11.3 \text{ V}}{608 \text{ k}\Omega} = 18.6 \mu\text{A} \end{aligned}$$

$$\begin{aligned} I_E &= (\beta + 1)I_B = (141)(18.6 \mu\text{A}) \\ &= 2.62 \text{ mA} \end{aligned}$$

$$r_e = \frac{26 \text{ mV}}{I_E} = \frac{26 \text{ mV}}{2.62 \text{ mA}} = \mathbf{9.92 \Omega}$$

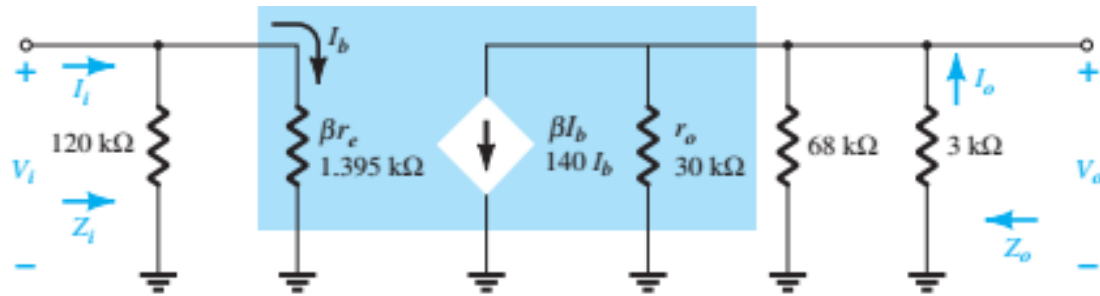
$$\text{b. } \beta r_e = (140)(9.92 \Omega) = 1.39 \text{ k}\Omega$$

The ac equivalent network appears in Fig. 5.53.

$$Z_i = R_{F1} \parallel \beta r_e = 120 \text{ k}\Omega \parallel 1.39 \text{ k}\Omega$$

$$\cong \mathbf{1.37 \text{ k}\Omega}$$

# Collector DC Feedback Configuration (Example)



**FIG. 5.53**

Substituting the  $r_e$  equivalent circuit into the ac equivalent network of Fig. 5.52.

c. Testing the condition  $r_o \geq 10R_C$ , we find

$$30 \text{ k}\Omega \geq 10(3 \text{ k}\Omega) = 30 \text{ k}\Omega$$

which is satisfied through the equals sign in the condition. Therefore,

$$\begin{aligned} Z_o &\cong R_C \parallel R_{F_2} = 3 \text{ k}\Omega \parallel 68 \text{ k}\Omega \\ &= \mathbf{2.87 \text{ k}\Omega} \end{aligned}$$

d.  $r_o \geq 10R_C$ ; therefore,

$$\begin{aligned} A_v &\cong -\frac{R_{F_2} \parallel R_C}{r_e} = -\frac{68 \text{ k}\Omega \parallel 3 \text{ k}\Omega}{9.92 \Omega} \\ &\cong -\frac{2.87 \text{ k}\Omega}{9.92 \Omega} \\ &\cong \mathbf{-289.3} \end{aligned}$$

e.  $|A_v| = 289.3 = \frac{V_o}{V_i}$

$$V_o = 289.3V_i = 289.3(2 \text{ mV}) = \mathbf{0.579 \text{ V}}$$



Thank You!

